

# Probabilistic Motion Planning in Uncertain and Dynamic Environments

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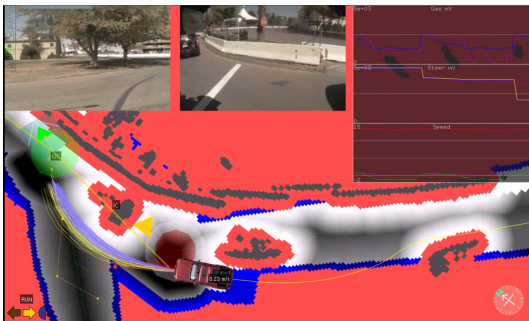
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September 26, 2017

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## The MIT-Cornell Collision in DARPA Urban Challenge 2017



1. Sensor data association;
2. Failure to anticipate vehicle's intention;
3. Overemphasis on lane constraints <sup>1</sup>.

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<sup>1</sup>L. Fletcher, S. Teller, E. Olson, *et al.*, "The MIT - Cornell collision and why it happened", *Springer Tracts in Advanced Robotics*, vol. 56, pp. 509–548, 2009

# Cooperative VS Non-cooperative

- Cooperative planning:  
All the agents mutually affect each other (interactions)
- Non-cooperative planning:  
The ego-vehicle takes all responsibility to avoid collision. → Freezing robot problem <sup>2</sup>.

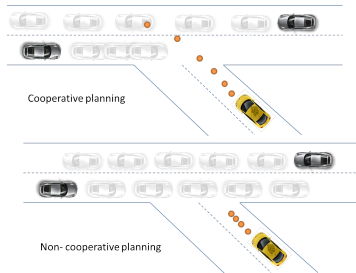


Figure : Cooperative planning and Non-cooperative planning

<sup>2</sup>P. Trautman and A. Krause, "Unfreezing the robot: Navigation in dense, interacting crowds", *IEEE/RSJ 2010 International Conference on Intelligent Robots and Systems, IROS 2010 - Conference Proceedings*, pp. 797-803, 2010

# Problem Statement

## Targeted problems

- How to model the interaction between the ego-vehicle and obstacles?
- How to incorporate the uncertainty of motion intentions into the trajectory planning?

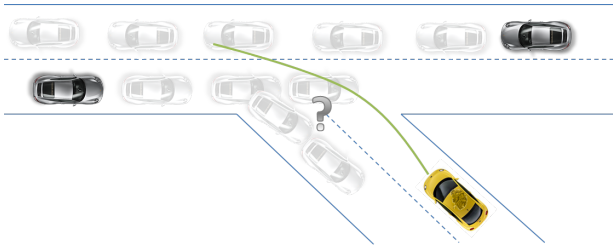


Figure : Where and How will the obstacle react?

# Problem Statement

- States:  $\mathbf{z}^R = [x, y, \theta, \delta, v]$ ,  $\mathbf{z}^A = [x, y, \theta, v, g]$ ,  $\mathbf{s} = [x, y]$ ,  
motion intention  $g \in \mathcal{G} = \{\text{Going straight, Turning, Stopping}\}$
- Controlled actions:  $\mathbf{u} = [u^\delta, u^a]$
- Trajectory of vehicle:  $\pi_i(g_j) = \{\mathbf{s}_0^i, \mathbf{s}_1^i, \dots, \mathbf{s}_m^i\}$
- Multipolicy under all motion intentions:  $\Pi^i = \{\pi_i(g_1), \dots, \pi_i(g_l)\}$
- Likelihood of motion intentions:  $\mathcal{W} = \{w_1, \dots, w_l\}$
- Optimization problem:

$$\begin{aligned} \mathbf{u}_{0:m-1}^* &= \min_{\mathbf{u}_{0:m-1}} \sum_{k=0}^{m-1} J(\mathbf{z}_k, \mathbf{u}_k) + J(\mathbf{z}_m) \\ \text{s.t. } \mathbf{z}_{k+1}^R &= f(\mathbf{z}_k^R, \mathbf{u}_k), & \forall k = \{0, \dots, m-1\} \\ \mathbf{z}_{\min}^R &< \mathbf{z}_k^R < \mathbf{z}_{\max}^R, & \forall k = \{0, \dots, m\} \\ \mathbf{u}_{\min} &< \mathbf{u}_k < \mathbf{u}_{\max}, & \forall k = \{0, \dots, m-1\} \\ &\text{Estimate } \Pi^i \text{ and } \mathcal{W}^i \text{ given } \pi^R, & \forall i = \{1, \dots, n\} \\ \mathbf{s}_k^R &\notin \mathbf{B}_k^i(\mathcal{W}^i, \Pi^i), & \forall i = \{1, \dots, n\}, \end{aligned} \tag{1}$$

# *Joint Behavior Estimation and Planning*

- Leverage the strengths of POMDP and MPC
- Chance constraint formulation for collision avoidance

## Assumptions:

- 1 The obstacle vehicles follow a reference path for each motion intention.
- 2 The boundary of road is given.
- 3 The reference path of the vehicle is the central line of the road.

# Joint Behavior Estimation & Planning - Overview

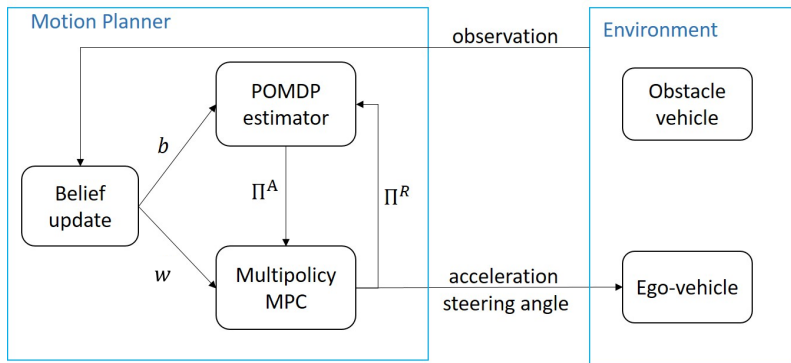


Figure : Scheme of joint behavior estimation and planning



# Behavior Estimation

## Aim:

Formulate a POMDP model to compute the optimal sequence of actions  $\Pi^A$  for obstacle vehicles under all motion intentions  $g$ .

Typical POMDP framework:

$$\{\mathbf{Z}, \mathbf{A}, \mathbf{O}, T(\mathbf{z}_{t+1}, \mathbf{a}_t, \mathbf{z}_t), O(\mathbf{o}_t, \mathbf{z}_{t+1}, \mathbf{a}_t), R(\mathbf{z}, \mathbf{a})\}$$

- State  $\mathbf{Z}$ :  $\mathbf{z}^R = [x, y, \theta]$ ,  $\mathbf{z}^A = [x, y, \theta, v, g]$
- Action  $\mathbf{A}$ :  $\mathbf{A} = \{\text{Acceleration, Deceleration, Maintain}\}$
- Observation  $\mathbf{O}$ :  $\mathbf{o}^R = [x, y, \theta]$ ,  $\mathbf{o}^A = [x, y, \theta]$

# Behavior Estimation

- Transition  $T(\mathbf{z}_{t+1}, \mathbf{a}_t, \mathbf{z}_t)$ : The ego-vehicle follows the computed policy from MPC planner (initially assume the constant velocity model).

$$\begin{bmatrix} x_{t+1} \\ y_{t+1} \\ \theta_{t+1} \\ v_{t+1} \end{bmatrix} = \begin{bmatrix} x_t \\ y_t \\ \theta_t \\ v_t \end{bmatrix} + \begin{bmatrix} (v_t + a\Delta t)\Delta t \cos(\theta_t) \\ (v_t + a\Delta t)\Delta t \sin(\theta_t) \\ \Delta\theta \\ a\Delta t \end{bmatrix}$$

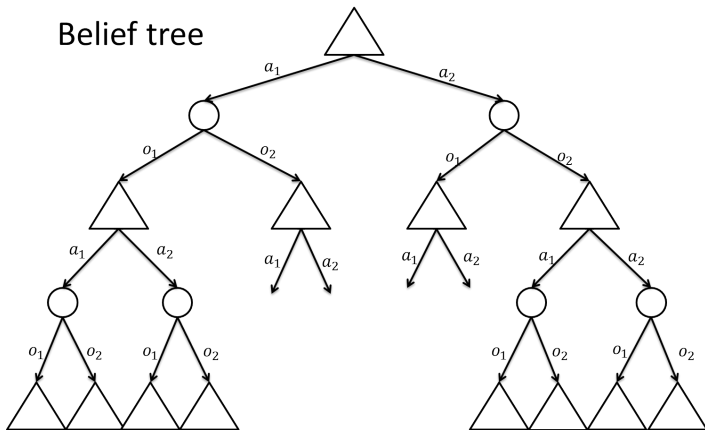
- Observation  $O(\mathbf{o}_t, \mathbf{z}_{t+1}, \mathbf{a}_t)$ : The poses of the vehicle are directly observable.
- Reward  $R(\mathbf{z}, a)$ :

$$R(\mathbf{z}, a) = R_{progress}(\mathbf{z}, a) + R_{collision}(\mathbf{z}, a) + R_{action}(a)$$

$$R_{collision} = \begin{cases} -r_c, & \text{if } \mathbf{s}^0 \in \mathbf{E} \\ 0, & \text{others} \end{cases}, \quad R_{action} = \begin{cases} -r_a, & \text{if } a \in [Acc, Dec] \\ 0, & \text{others} \end{cases},$$
$$R_{progress} = \beta e^{-\frac{d}{2\sigma^2}}$$

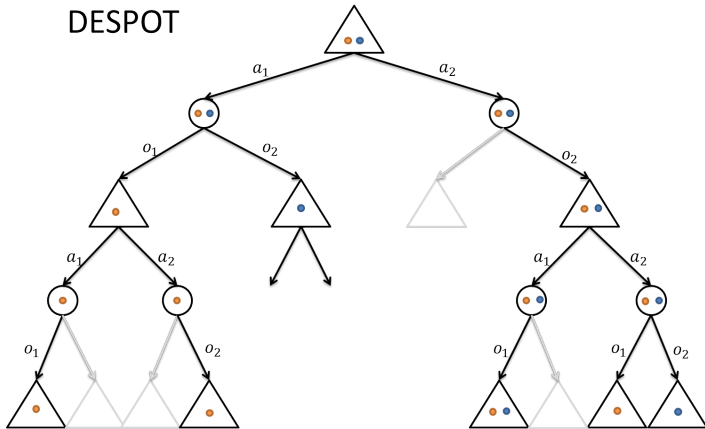
# Behavior Estimation

## POMDP Solver - DESPOT<sup>3</sup>

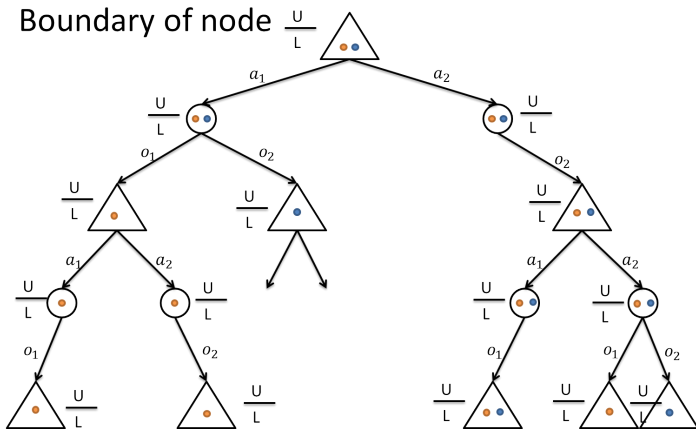


<sup>3</sup>A. Somani, N. Ye, D. Hsu, and W. S. Lee, "DESPOT : Online POMDP Planning with Regularization", *Advances in Neural Information Processing Systems*, pp. 1–9, 2013

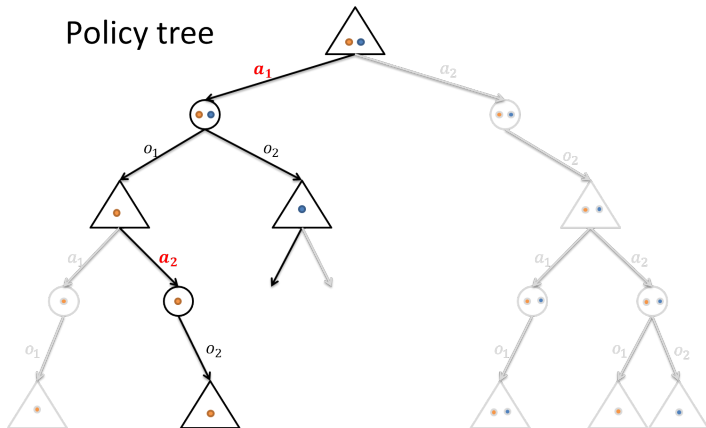
# Behavior Estimation



# Behavior Estimation



# Behavior Estimation

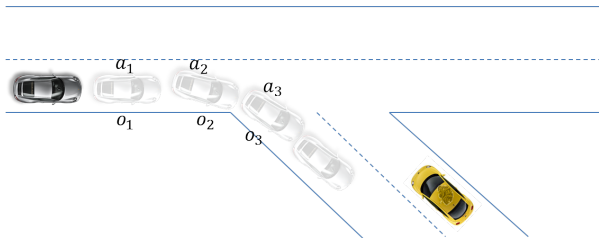
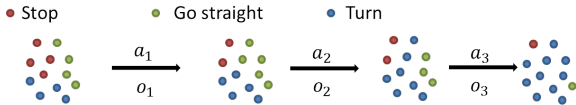


$$V(b) = \max_{a \in A} \left\{ \sum_{z \in Z} b(z) R(z, a) + \gamma \sum_{o \in O} p(o|b, a) V(\tau(b, a, o)) \right\}$$

# Behavior Estimation

Belief update (Particle filter)

$$b_{t+1} = \eta O(\mathbf{o}_t, \mathbf{z}_{t+1}, \mathbf{a}_t) \sum_{\mathbf{z} \in \mathcal{Z}} T(\mathbf{z}_{t+1}, \mathbf{a}_t, \mathbf{z}_t) b_t$$



$$w_j = \frac{\text{number of particles with motion intention } g_j}{\text{overall number of particles}}$$

# Receding Horizon Trajectory Planning

## Aim:

Compute the optimal trajectory  $\mathbf{u}_{0:m-1}$  of the ego-vehicle using the obstacles' estimated trajectories  $\Pi^A$  and associated beliefs  $w_j$  of intentions

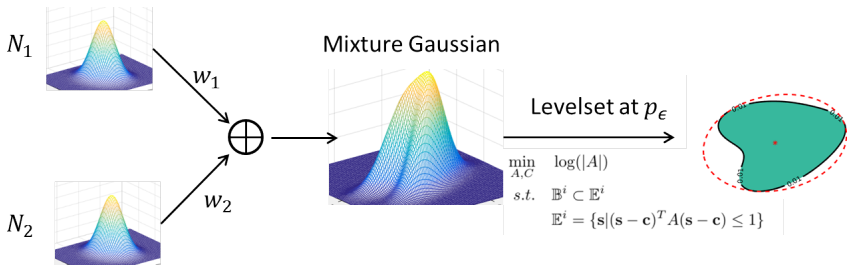
$$\begin{aligned} \min_{\mathbf{u}_{0:m-1}} & \sum_{k=0}^{m-1} J(\mathbf{z}_k^R, \mathbf{u}_k, \lambda_k) \Delta t_k + J(\mathbf{z}_m^R, \lambda_m) \Delta t_k \\ \text{s.t.} & \quad \mathbf{z}_{k+1}^R = f(\mathbf{z}_k^R, \mathbf{u}_k) \\ & \quad \lambda_{k+1} = \lambda_k + v_k \Delta t_k \\ & \quad \mathbf{z}_{min}^R < \mathbf{z}_k^R < \mathbf{z}_{max}^R \\ & \quad \mathbf{u}_{min} < \mathbf{u}_k < \mathbf{u}_{max} \\ & \quad b_l(\lambda_k) + w_{max} \leq d(\mathbf{z}_k, \lambda_k) \leq b_r(\lambda_k) - w_{max} \\ & \quad \mathbf{s}_k^0 \notin \mathbf{E}_k^i(\mathcal{W}^i, \Pi^i) \quad \forall i = \{1, \dots, n\} \\ & \quad \forall k = \{1, \dots, m-1\} \end{aligned}$$



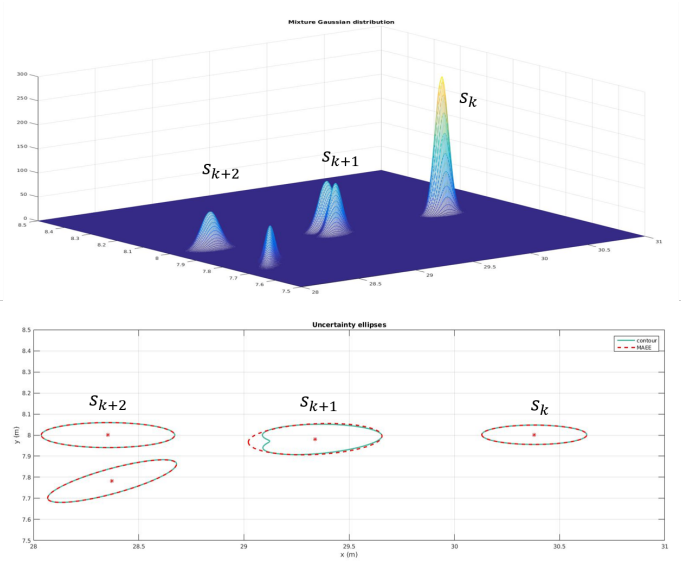
# Receding Horizon Trajectory Planning

## Probabilistic Collision Constraint:

Compute the uncertainty ellipses for each obstacle vehicle regarding under a certain collision probability  $p_\epsilon$  regarding the multiple policies  $\Pi^A$  and associated weights  $\mathcal{W}$



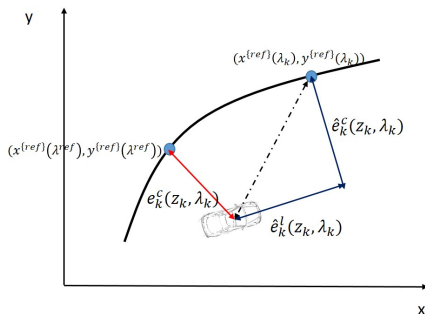
# Receding Horizon Trajectory Planning



# Receding Horizon Trajectory Planning

- Cost function:

$$J = \mathbf{p}^T \begin{bmatrix} \|\hat{\mathbf{e}}_k^c(x_k, y_k, \lambda_k)\|^2 \\ \|\hat{\mathbf{e}}_k^l(x_k, y_k, \lambda_k)\|^2 \\ v_k \Delta t_k \\ \|u_k^a\|^2 \\ \|u_k^\delta\|^2 \\ \|\dot{\theta}_k\|^2 \end{bmatrix}$$



- Dynamics:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\delta} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} v \cos(\theta) \\ v \sin(\theta) \\ \frac{v}{L} \tan(\delta) \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u^\delta \\ u^a \end{bmatrix}$$

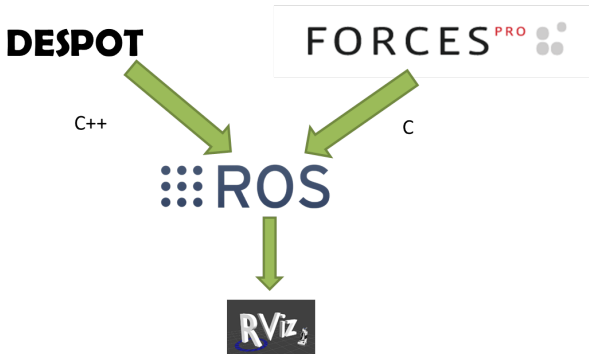


Figure : Simulation platform

## Negotiation with 4 obstacles in T-junction scenario

### Simulation in T-junction Scenario

Task:

Navigate the ego-vehicle to safely merge to the traffic with the negotiation of 4 obstacle vehicles.

# Results

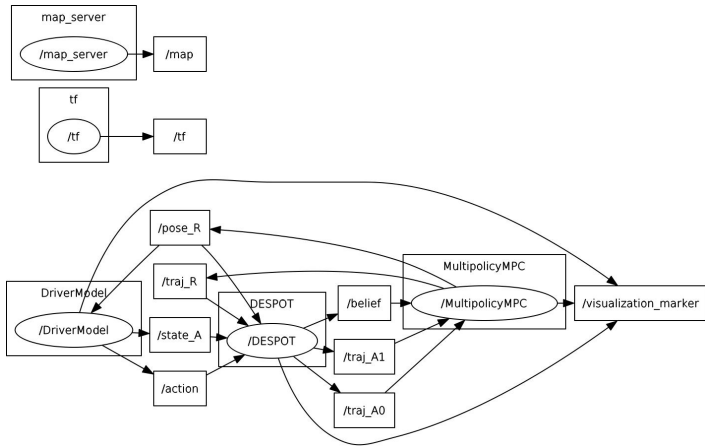
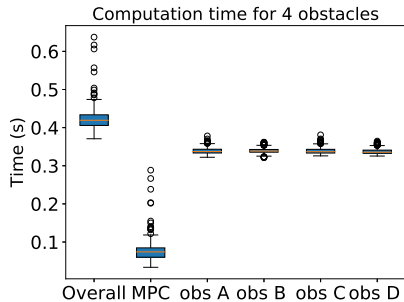
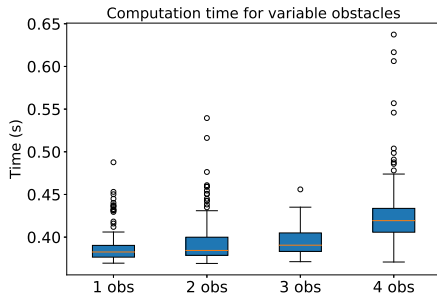


Figure : ROS graph of simulation

# Results



(a) 4 obstacles



(b) Variable obstacles

Figure : Computation time for our method

Table : Comparison with state-of-the-art

Approach	Real time	Scalability	Safety	Non-holonomic Model
Our approach	✓	✓ ✓	✓ ✓	✓ ✓
IGP <sup>4</sup>	✓ ✓	✓ ✓	✓	✓ <sup>1</sup>
Multipolicy <sup>5</sup>	✓ ✓ ✓	✓	✓ ✓	✓ ✓ <sup>2</sup>
Online POMDP <sup>6</sup>	✓	✓ ✓	✓ ✓	✓

- 1: Gaussian process as the dynamic model
- 2: Ego-vehicle's motion is restricted in the predefined policy sets

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<sup>4</sup>P. Trautman, J. Ma, R. M. R. M. Murray, and A. Krause, "Robot Navigation in Dense Human Crowds: the Case for Cooperation", *Cds.Caltech.Edu*, pp. 2153–2160, 2013

<sup>5</sup>E. Galceran, A. G. Cunningham, R. M. Eustice, and E. Olson, "Multipolicy decision-making for autonomous driving via changepoint-based behavior prediction", in *Proceedings of Robotics: Science and Systems (RSS)*, Rome, Italy, 2015

<sup>6</sup>W. Liu, S. W. Kim, S. Pendleton, and M. H. Ang, "Situation-aware decision making for autonomous driving on urban road using online POMDP", in *IEEE Intelligent Vehicles Symposium, Proceedings*, vol. 2015-August, 2015, pp. 1126–1133



# Conclusion and Future Work

## Contributions:

- A joint behavior estimation and trajectory planning method, utilizing the strengths of MPC and online POMDPs to achieve intention-aware navigation.
- A chance constrained formulation of MPC accounting for the uncertainty in the motion intentions of other traffic participants, over multiple motion policies.

## Future work:

- The real time capability of the estimation needs to be improved.
- The uncertainty of the dynamics needs to be considered for the obstacle vehicle's model.
- It would be highly interesting to test it in a mobile robot.

*Thanks! Q & A*