Adaptive Path Following for UAV in Time Varying Unknown Wind Environments

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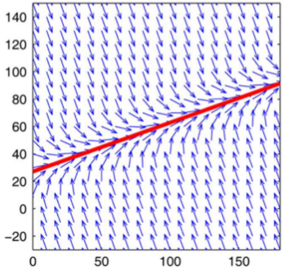
May 24, 2017

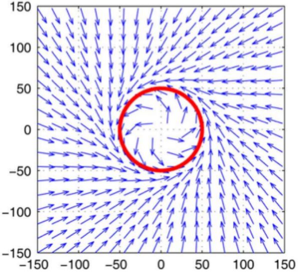
Outline

Introduction

Vector Field (VF) Path Following

Given a reference path $R^2(R^3)$, build up a group of vectors around the reference path as the control inputs (steering angle, speed) to the UAV so that it can converge to the reference path asymptotically.





Weakness:

Standard VF only works for known, constant wind disturbance

Why not using Adaptive Control?

- Compensate the wind disturbance
- Limit the path following error at least bounded

Kinematics and Control Task

- UAV kinematics in 2D

$$\begin{aligned} \dot{x} &= V_a \cos\psi + W\cos\psi_w + A\cos\psi_A \\ \dot{y} &= V_a \sin\psi + W\sin\psi_w + A\sin\psi_A \end{aligned} \quad (1)$$

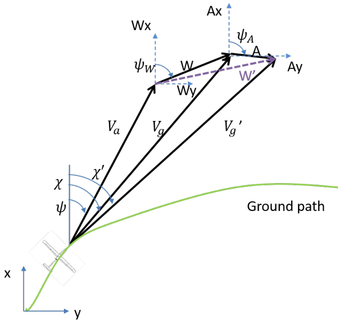


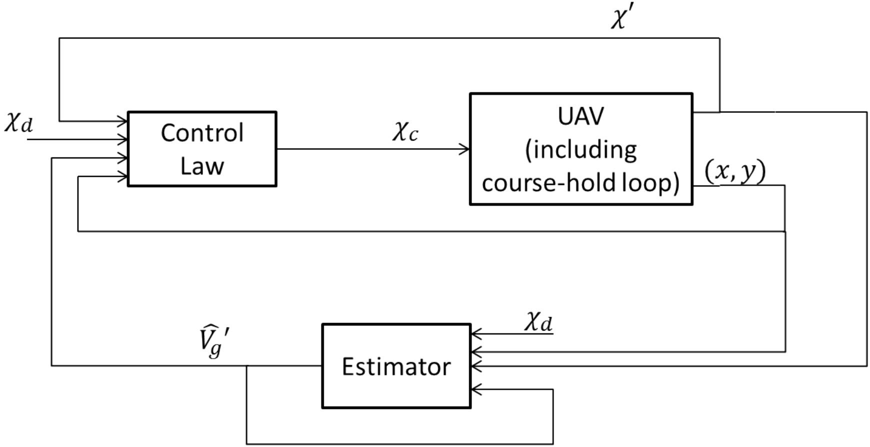
Figure: UAV kinematics

x, y: position of UAV
V_a: airspeed of UAV
ψ: heading angle between airspeed and horizontal axis
χ': UAV's course angle
W: Constant wind amplitude
A: Time Varying wind amplitude
ψ_w: angle of constant wind in earth frame
ψ_A: angle of time varying wind in earth frame

Kinematics and Control Task

- Assumptions
  1. Altitude and airspeed (V_a) are held constant by the longitudinal control of UAV;
  2. The UAV is equipped with the course-hold loop devices whose dynamics can be modeled as the first-order system
  $$\dot{\chi}' = \alpha(\chi_c - \chi')$$
  3. The UAV course is measurable;
  4. A slowly time-varying unknown component of wind with amplitude $A(t)$ and angle $\psi_A(t)$.
- Control task
  Build up the control law χ_c to let the UAV follow the path as accurately as possible under the wind disturbance.

Kinematics and Control Task



Path Following Strategies

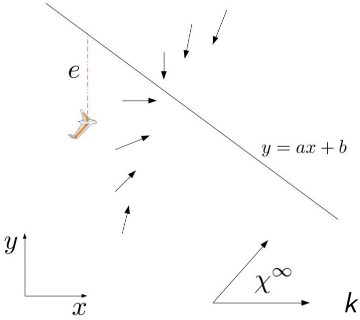
Adaptive Vector Field Path Following Strategy:

- Straight Line Following
- Orbit Path Following

Straight Line Following

Task

Find the control law which can steer the UAV to the reference straight line and keep along with the path.



1. Distance error

$$e = y - (ax + b)$$

2. Course error

$$\tilde{\chi}' = \chi' - \chi_d$$

3. Desired course

$$\chi_d = -\chi^\infty \frac{2}{\pi} \tan^{-1}(ke) + \tan^{-1}(a)$$

k: a positive constant influences the rate of course transition from χ^∞ to $\tan^{-1}(a)$.

Straight Line Following

- If $\chi' \to \chi_d$, distance error will converge to zero.

Proof.

Lyapunov function $\mathcal{V}_1 = \frac{1}{2}e^2$

$$\begin{aligned}\dot{\mathcal{V}}_1 &= e(\dot{y} - a\dot{x}) \\ &= eV_g'(\sin\chi_d - a\cos\chi_d) \\ &= eV_g'\frac{\sin(i\chi^{\infty}\frac{2}{\pi}\tan^{-1}(ke))}{\cos(\tan^{-1}a)} < 0\end{aligned}$$

$\square$

Straight Line Following

- Then derive the control law of the course angle
  Define the Lyapunov function $\mathcal{V}_2 = \frac{1}{2}\tilde{\chi}'^2$

$$
\begin{aligned}
\dot{\mathcal{V}}_2 &= \tilde{\chi}'\dot{\tilde{\chi}}' \\
&= \tilde{\chi}'(\alpha(\chi_c - \chi') + \chi^\infty \frac{2}{\pi}\frac{k\dot{e}}{1+(ke)^2}) \\
&= \tilde{\chi}'(\alpha(\chi_c - \chi') + \chi^\infty \frac{2}{\pi}\frac{k}{1+(ke)^2} V_g'(\sin\chi' - a\cos\chi'))
\end{aligned}
$$

Ideally, if we choose the command course as

$$
\chi_c = \chi' - \frac{1}{\alpha}\chi^\infty \frac{2}{\pi}\frac{k}{1+(ke)^2} V_g'(\sin\chi' - a\cos\chi') - \frac{\kappa}{\alpha} sat(\frac{\tilde{\chi}'}{\epsilon}) \quad (2)
$$

$\kappa > 0$: the shape of the trajectories on the sliding surface;
$\epsilon > 0$: the width of the transition region at the sliding surface.

The derivative of Lyapunov function is negative semi-definite.

Straight Line Following

Unfortunately, the control law (Eq.(2)) can not be implemented directly!!

$$\chi_c = \chi' - \frac{1}{\alpha}\chi^{\infty}\frac{2}{\pi}\frac{k}{1+(ke)^2}V_g'(\sin\chi' - a\cos\chi') - \frac{\kappa}{\alpha}sat(\frac{\tilde{\chi}'}{\epsilon})$$

Straight Line Following

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$$\chi_c = \chi' - \frac{1}{\alpha}\chi^{\infty}\frac{2}{\pi}\frac{k}{1+(ke)^2}V_g'(\sin\chi' - a\cos\chi') - \frac{\kappa}{\alpha}sat(\frac{\tilde{\chi}'}{\epsilon})$$

We need ESTIMATOR for the ground velocity V_g'

$$\chi_c = \chi' - \frac{1}{\alpha}\chi^{\infty}\frac{2}{\pi}\frac{k}{1+(ke)^2}\hat{V}_g{}'(\sin\chi' - a\cos\chi') - \frac{\kappa}{\alpha}sat(\frac{\tilde{\chi}'}{\epsilon}) \quad (3)$$

Straight Line Following

Time to design the estimator for ground velocity.

Straight Line Following

Theorem

In straight line following scenario, the command course (Eq.(3)) and the estimator

$$\dot{\hat{V}}_g' = \Gamma \rho \tilde{\chi}' \chi^{\infty} \frac{2}{\pi} \frac{k}{1 + (ke)^2} (\sin \chi' - a \cos \chi') - \sigma \Gamma \hat{V}_g{}' \tag{4}$$

($\Gamma > 0$: the estimation gain, $\sigma > 0$: a switching σ-modification parameter.)
guarantees that the tracking error converges to zero for unknown constant winds and stays bounded for unknown slowly time-varying wind.

Straight Line Following

Proof:
Define the estimator error as $\Theta = \hat{V_g}' - V_g'$. The derivative of Lyapunov function $\mathcal{V}_e = \mathcal{V}_1 + \rho\mathcal{V}_2 + \frac{1}{2}\Gamma^{-1}\Theta^2$ is

$$\begin{aligned}\dot{\mathcal{V}}_e &= \dot{\mathcal{V}}_1 + \rho\dot{\mathcal{V}}_2 + \Gamma^{-1}\Theta\dot{\Theta} \\ &= \dot{\mathcal{V}}_1 + \rho\tilde{\chi}'[-\chi^\infty \frac{2}{\pi}\frac{k}{1+(ke)^2}(\hat{V_g}' - V_g')(\sin\chi' - a\cos\chi') \\ &\quad - \kappa sat(\frac{\tilde{\chi}'}{\epsilon})] + \Gamma^{-1}(\hat{V_g}' - V_g')(\dot{\hat{V}}_g' - \dot{V_g}')\end{aligned}$$

ρ: positive weight term for course error, which is aimed to make the distance error and course error compatible.

Straight Line Following

First, we prove the tracking errors (e and $\tilde{\chi}'$) will converge to zero under the assumption that $\dot{V_g}' = 0$.

$$\dot{\mathcal{V}}_e = \dot{\mathcal{V}}_1 + \rho\tilde{\chi}'[-\chi^{\infty}\frac{2}{\pi}\frac{k}{1+(ke)^2}(\hat{V_g}' - V_g')(\sin\chi' - a\cos\chi') - \kappa sat(\frac{\tilde{\chi}'}{\epsilon})]\times$$

$$+ \Gamma^{-1}(\hat{V_g}' - V_g')\dot{\hat{V}}_g'$$

$$= \dot{\mathcal{V}}_1 - \rho\kappa\tilde{\chi}' sat(\frac{\tilde{\chi}'}{\epsilon}) + \{\dot{\hat{V}}_g'\Gamma^{-1} - \rho\tilde{\chi}'\chi^{\infty}\frac{2}{\pi}\frac{k}{1+(ke)^2}(\sin\chi' - a\cos\chi')\}\times$$

$$(\hat{V_g}' - V_g')$$

If the estimator is chosen as

$$\dot{\hat{V}}_g' = \Gamma\rho\tilde{\chi}'\chi^{\infty}\tfrac{2}{\pi}\tfrac{k}{1+(ke)^2}(\sin\chi' - a\cos\chi')$$

Then the derivative of $\mathcal{V}_e$ is negative semi-definite.

$$\dot{\mathcal{V}}_e = \dot{\mathcal{V}}_1 - \rho\kappa\tilde{\chi}' sat(\frac{\tilde{\chi}'}{\epsilon})$$

Straight Line Following

However, $\dot{\mathcal{V}}_e < 0$ is not enough to prove the asymptotic convergence of tracking errors to zero for time varying systems.
Barbalat's Lemma for stability analysis of Time varying systems

$$\begin{aligned}
\ddot{\mathcal{V}}_e &= \ddot{\mathcal{V}}_1 - \rho\kappa sat(\frac{\tilde{\chi}'}{\epsilon})\dot{\tilde{\chi}}' \\
&= \ddot{\mathcal{V}}_1 - \rho\kappa sat(\frac{\tilde{\chi}'}{\epsilon})[-\chi^{\infty}\frac{2}{\pi}\frac{k}{1+(ke)^2}(\sin\chi' - a\cos\chi')\Theta \\
&\quad - \kappa sat(\frac{\tilde{\chi}'}{\epsilon})]
\end{aligned}$$

$\ddot{\mathcal{V}}_e$ is bounded, $\dot{\mathcal{V}}_e <= 0$, $\mathcal{V}_e >= 0 \Rightarrow \dot{\mathcal{V}}_e \to 0$ as $t \to \infty$.
Conclusion: e and $\tilde{\chi}'$ converge to zero asymptotically.

Straight Line Following

Then, we prove the tracking errors will be bounded for unknown *slowly time-varying* wind by using the σ-modification technique.

$$\dot{\mathcal{V}} = -\rho\kappa\tilde{\chi}' sat(\frac{\tilde{\chi}'}{\epsilon}) + \{(\dot{\hat{V}}_g' - \dot{V_g}')\Gamma^{-1} - \rho\tilde{\chi}'\chi^{\infty}\frac{2}{\pi}\frac{k}{1+(ke)^2}\times$$
$$(\sin\chi' - a\cos\chi')\}(\hat{V_g}' - V_g')$$
$$= -\rho\kappa\tilde{\chi}' sat(\frac{\tilde{\chi}'}{\epsilon}) - \sigma\Theta^2 - \sigma\Theta(-\Gamma^{-1}\dot{V_g}'\sigma^{-1} - V_g')$$

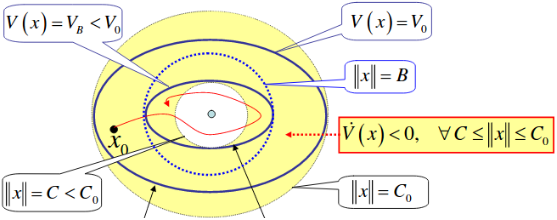
Using the inequality $-a^2 + ab \leq -\frac{a^2}{2} + \frac{b^2}{2}$ for any a and b, we write

$$\dot{\mathcal{V}} \leq -\rho\kappa\tilde{\chi}' sat(\frac{\tilde{\chi}'}{\epsilon}) - \frac{\sigma}{2}\Theta^2 + \frac{\sigma(V_g' + \dot{V_g}'\Gamma^{-1}\sigma^{-1})^2}{2}$$
$$= -\rho\kappa\tilde{\chi}' sat(\frac{\tilde{\chi}'}{\epsilon}) - \frac{\sigma}{2}\Theta^2 + constant$$

If $\Theta^2 \geq \frac{2C}{\sigma}$, $\dot{\mathcal{V}}$ will be negative definite $\Rightarrow$ e, $\tilde{\chi}'$ and Θ will converge inside a ball around the origin and stay bounded.

Straight Line Following

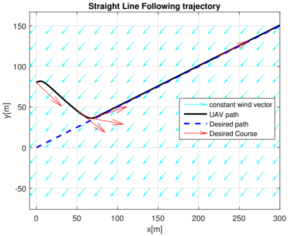
- Uniform Ultimate Boundedness
  The solution of $\dot{x} = f(x,t)$ starting at $x(t_0) = x_0$ are Uniformly Ultimately Bounded (UUB) with ultimate bound B if:
  $\exists C_0 > 0, T = T(C_0, B) > 0 : (\|x(t_0)\| \leq C_0) \Rightarrow (\|x(t)\| \leq B, \forall t \geq t_0 + T.$



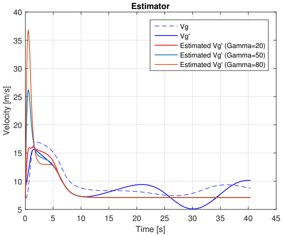
All trajectories starting in large ellipse enter small ellipse within finite time $T(C_0, B)$.

Simulation Results

- Performance of controller and estimator

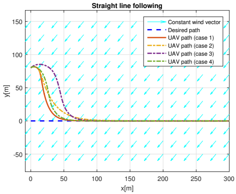


(a) Straight line following performance



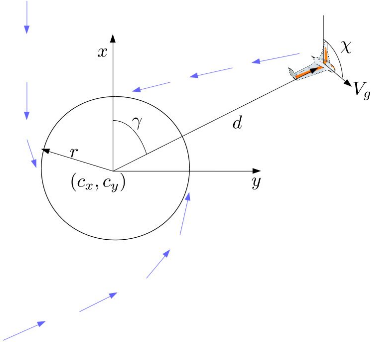
(b) Estimation performance

- Effect of design parameters



Influence of design parameters for straight line following:
Case 1: $k = 0.1, \kappa = \frac{\pi}{2}, \epsilon = 0.5, \Gamma = 50;$
Case 2: $k = 0.05, \kappa = \frac{\pi}{2}, \epsilon = 0.5, \Gamma = 50;$
Case 3: $k = 0.1, \kappa = \frac{\pi}{6}, \epsilon = 0.5, \Gamma = 50;$
Case 4: $k = 0.1, \kappa = \frac{\pi}{2}, \epsilon = 1.5, \Gamma = 50.$

Orbit Path Following



- UAV kinematics in the polar coordinate

$$\dot{d} = V_g' \cos(\chi' - \gamma)$$
$$\dot{\gamma} = \frac{V_g'}{d} \sin(\chi' - \gamma)$$

- Distance error

$$\tilde{d} = d - r$$

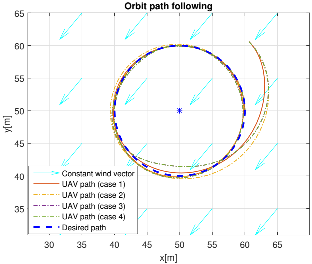
- Course error

$$\tilde{\chi}' = \chi' - \chi_d$$

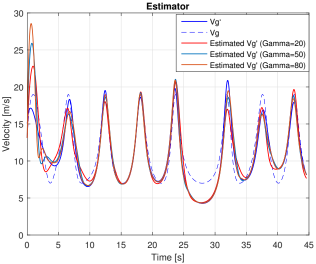
- Desired course

$$\chi_d = \gamma - \left[\frac{\pi}{2} + \tan^{-1}(k\tilde{d})\right]$$

Simulation Results

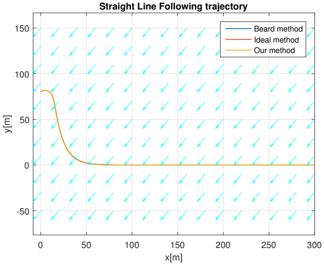


(a) Influence of design parameters for orbit following:
Case 1: $k = 0.1, \kappa = \frac{\pi}{2}, \epsilon = 0.5, \Gamma = 50$;
Case 2: $k = 0.05, \kappa = \frac{\pi}{2}, \epsilon = 0.5, \Gamma = 50$;
Case 3: $k = 0.1, \kappa = \frac{\pi}{6}, \epsilon = 0.5, \Gamma = 50$;
Case 4: $k = 0.1, \kappa = \frac{\pi}{2}, \epsilon = 1.5, \Gamma = 50$.

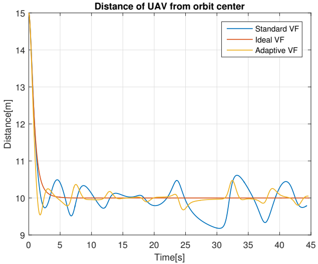


(b) Estimation performance

Comparison



(a) Straight line following



(b) Orbit path following

Table: Steady state RMS error for straight line following

	Std. VF	Id. VF	Adap. VF
RMS	0.2203	0.1573	0.1434

Table: Steady state RMS error for orbit following

	Std. VF	Id. VF	Adap. VF
RMS	0.33	6.08×10^{-6}	0.1219

Thanks for your attention!